Billiards on flat thorus and Lorenz models

Prof. Radu Nikolae Gologan (Institute of Mathematics of the Romanian Academy, Romania)

Let $\ell \geqslant 2$ be a positive integer. Consider the set $\mathbb{Z}^2_{(\ell)}$ of pairs (m, n) such that $m - n$ is not 0 modulo ℓ. For $\varepsilon > 0$ consider scaterers of radius ε centered at $\mathbb{Z}^2_{(\ell)}$ and the region

$$Z_{\ell,\varepsilon} = \{x \in \mathbb{R}^2 : \mathrm{dist}(x, \mathbb{Z}^2_{(\ell)}) \geqslant \varepsilon\}$$

obtained by eliminating the scaterers. Consider in $\mathbb{R}^2$ the free rectilinear of unit speed of a particle starting at $(0, 0)$. The first exit time in the direction ω is defined by

$$\tau_{\ell,\varepsilon}(\omega) = \inf\{\tau > 0 : \tau\vec{\omega} \in \partial Z_{\ell,\varepsilon}\},$$

and corresponds to the distance to the first scaterer intersected by the $\vec{\omega} = e^{i\omega} \in \mathbb{T}$ array, and is $+\infty$ when the particle escapes to infinity. The paper is devoted to the limit when $\varepsilon \to 0$, of the repartition function $\varepsilon\tau_{\ell,\varepsilon}$ defined by

$$\mathbb{P}_{\ell,\varepsilon}(\lambda) = \frac{1}{2\pi}\left|\left\{\omega \in [0, 2\pi] : \tau_{\ell,\varepsilon}(\omega) > \frac{\lambda}{\varepsilon}\right\}\right|.$$

All results are exact and are proved using distribution theorems for Farey numbers using the harmonical analysis of Kloosterman sums. This is a joint paper with Florin Boca and is based of a series of papers on the subject by the author, Florin Boca and Alexandru Zaharescu.